$\bar{u} = 3/2\ [1 - y^2]$ = dimensionless mean velocity profile, $\bar{u}^*/u_m$
u_m = average velocity in the film, cm./sec.
x = dimensionless Cartesian coordinate in direction of flow, x^*/h
y = dimensionless Cartesian coordinate normal to the plate, y^*/h
α = dimensionless wave number, $2\pi h/\lambda$
λ = wavelength, cm.
ν = kinematic viscosity, sq.cm./sec.
ρ = density, g./cc.
σ = surface tension, dyne/cm.
$\phi(y)$ = dimensionless amplitude of disturbance stream function

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Manuscript received January 27, 1966; revision received April 25, 1966; paper accepted April 26, 1966.

Liquid Surface Area Effects in a Wetted-Wall Column

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Vaporization of methanol and carbon tetrachloride into air and carbon tetrachloride into helium was carried out in a counterflow wetted-wall column at 10° to 20°F. These data show an influence of liquid rate upon the mass transfer coefficients. Liquid rate influence has been empirically accounted for by a function of surface area increase caused by rippling of the liquid film.

Kafesjian et al. (*5, 6*) showed that the liquid flow rate must be considered as a variable in the correlation of gas phase mass transfer in wetted-wall columns. Except for very low liquid Reynolds numbers, thin films do not flow smoothly down a vertical wall. The surface becomes covered with ripples, increasing the interfacial contact area; in fact four different flow regimes have been defined (*2, 4, 11, 19*). Most of the data concerning wetted-wall correlations and performance whch were published prior to 1960 have been reviewed previously (*5, 6*).

More recently, Parrish (*10*) obtained data by vaporizing methanol into nitrogen and also into helium. Sebulsky (*14, 18*) vaporized benzene into nitrogen and also the binary benzene-acetone into helium. Strumillo and Porter (*15*) showed that the liquid rate increased the rate of mass transfer for carbon tetrachloride vaporizing in a wetted-wall column. Pertinent studies of thin liquid films in rippling flow have been published by Portalski et al. (*11, 12, 16, 17*). An excellent review of phenomena associated with flow of liquids in thin films has been given by Fulford (*3a*).

EXPERIMENTAL APPARATUS AND PROCEDURE

In the present investigation data have been obtained on low-temperature (10° to 20°F.) vaporization of methanol and carbon tetrachloride into air and carbon tetrachloride into helium in a counterflow wetted-wall column. This apparatus was similar to that described by Kafesjian (*6*). Complete details can be found in reference *13*. Temperatures of the inlet and exit gas and liquid streams were measured by fine wire copper-constantan thermocouples to an accuracy of ± 0.05°F. Exit compositions of the gas stream were measured by a thermal conductivity cell for the carbon tetrachloride runs. Wet and dry bulb thermometry was used for the methanol.

CALCULATION PROCEDURE

The mass transfer coefficient may be properly defined for small rates of mass transfer in a differential section by the equation

$$\frac{G}{M_{\text{avg}}\,P}\,dp = k_G a(p^* - p)\,dZ \tag{1}$$

or

$$\int_1^2 \frac{M_{\text{avg}} P\, k_G a\, dZ}{G} = \int_1^2 \frac{dp}{p^* - p} \tag{1a}$$

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Attention must be given to the evaluation of the integral on the right-hand side of Equation (1*a*). If the liquid remains at constant temperature, p^* is constant and simple integration and rearrangement yield a logarithmic mean driving force at the two ends of the column. Data of most investigators have shown that the liquid temperature varies. Accordingly, some authors have used a value of p^* evaluated at the arithmetic average of the inlet and outlet liquid temperatures, while others have used specific values of p^* at each end of the column appropriate to the liquid temperature at that position. Neither of these procedures is mathematically correct. It is necessary to consider the simultaneous transfer of heat and mass throughout the column. The technique used here is based on the following equations:

Mass transfer:

$$dW = k_G M \, (p^* - p) dA \tag{2}$$

Heat balance (gas):

$$G \frac{\pi \, d^2}{4} C_G \, dt_G = h_G (t_L - t_G) dA \tag{3}$$

Heat balance (liquid):

$$LC_L dt_L = \lambda \, dW + h_G (t_L - t_G) dA + U (t_L - t_f) dA \tag{4}$$

These equations were solved by using a modified Euler technique (*8*) with the aid of an IBM 1620 computer. In this procedure the various transfer coefficients were assumed and a step-by-step calculation was made from the bottom to the top of the tower. The calculated values of concentration and temperatures at the top of the tower were then compared with the experimental values. Transfer coefficients were changed and the iteration continued until calculated and measured values were in agreement. Values of k_G calculated in this manner deviated from those calculated with a logarithmic driving force by as much as 10%.

RESULTS AND DISCUSSION

Data for the air–carbon tetrachloride system are shown in Figure 1. Similar data were obtained for the other systems. These data show that liquid flow conditons affect the mass transfer. Cross plots indicate that an exponent of 0.83 satisfactorily accounts for the effect of gas phase Reynolds number. The value of 0.83 has been found valid by many authors. The value of 0.68 recently reported (*15a*) for vaporization of carbon tetrachloride into air appears to be lower than usually found, a fact noted by the authors themselves.

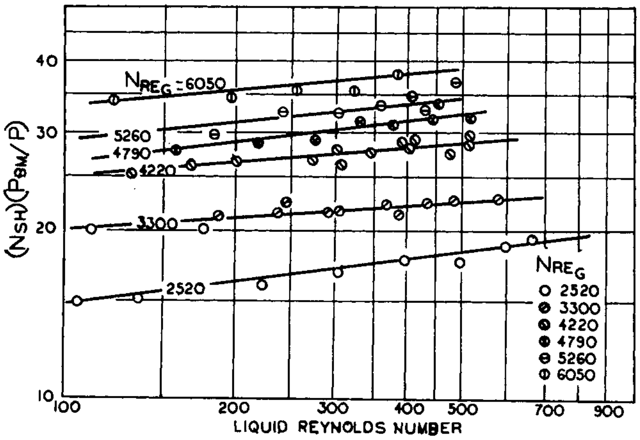

Fig. 1. Vaporization of carbon tetrachloride by air.

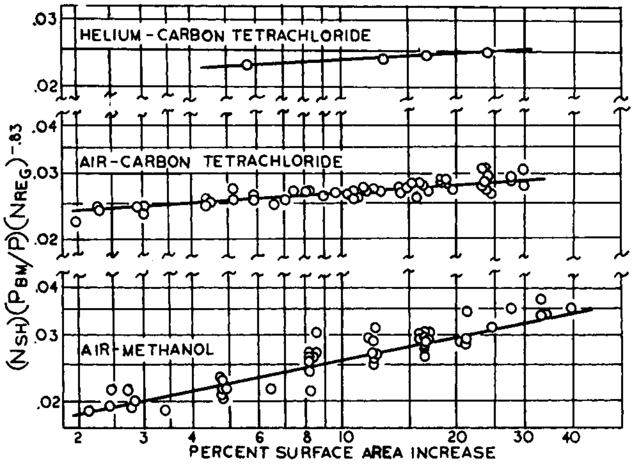

Fig. 2. Effect of surface area increase on vaporization mass transfer.

In vaporization of pure liquids, the most significant effect of surface rippling would appear to be an increase in surface area. Starting with the theoretical development of Kapitsa (*7*) Portalski (*11, 16*) made a number of simplifying assumptions and derived an equation relating the fractional increase of surface area by rippling to flow conditions and physical properties. For liquid Reynolds numbers of ten or greater, Portalski's equation can be transformed into:

$$\Delta S = 0.721 (N_{Re_L})^{5/3} (g \, \nu_L)^{1/3} \frac{\mu_L}{\sigma} \tag{5}$$

Based on this concept the Sherwood number divided by the gas phase Reynolds number to the 0.83 power was plotted against the percent surface area increased as shown in Figure 2. The data for all three systems showed less scatter when plotted in this manner than when plotted vs. the liquid Reynolds number. Lines in Figure 2 are represented by the following equations calculated by the method of least squares:

Helium-carbon tetrachloride:

$$(\text{MTG}) = 0.0204 \, (\Delta S)^{0.064} \tag{6}$$

Air-carbon tetrachloride:

$$(\text{MTG}) = 0.0224 \, (\Delta S)^{0.072} \tag{7}$$

Air-methanol:

$$(\text{MTG}) = 0.0159 \, (\Delta S)^{0.206} \tag{8}$$

For convenience the term $N_{Sh} (N_{Re_G})^{-0.83} \left(\frac{P_{BM}}{P} \right)$ is referred to as the mass transfer group (MTG). While the exponent in Equations (6), (7), and (8) varies from system to system, the same trend is noticed when the liquid Reynolds number is used as a variable. Equation (7) also correlated data taken at room temperature (*5*), indicating no specific temperature effect. The above equations should be valid for $100 < N_{Re_L} < 800$.

When the surface area is constant or known the gas phase Schmidt number is normally used to correlate data for various systems. The molecular weights of the carrier gas and vaporizing liquid are widely different; therefore a gas phase Schmidt number evaluated at film log-mean conditions was used. Values of $(\text{MTG})/(\Delta S)^\epsilon$ were plotted as a function of the Schmidt number. The present results along with the data of Kafesjian (*5*), for which $\epsilon = 0.125$, are shown in Figure 3. A similar plot with a Schmidt number evaluated at bulk conditions showed more scatter. A line with a slope of 0.5 is a good representation of the data shown in Figure 3. This procedure

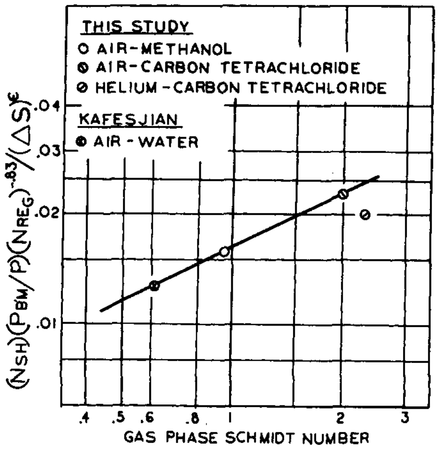

Fig. 3. Wetted-wall mass transfer correlation.

evaluates the mass transfer effects at conditions where liquid rates and hence ΔS is small. Under these circumstances an exponent of 0.5 on the Schmidt number seems reasonable and is more in agreement with mass transfer theory than other exponents frequently used. The resulting equation which correlates the four sets of data is

$$N_{Sh}\left(\frac{p_{BM}}{P}\right) = 0.163\ (N_{ReG})^{0.83}(N_{ScG})^{0.5}(\Delta S)^{\epsilon} \quad (9)$$

where ϵ takes on different values for each system.

There appears to be some disagreement in the literature (*3a*) as to the amount of surface area increase caused by wavy flow. Some authors suggest that Portalski's equations predict values that are too large. The small values of ϵ noted in this work may lend some credence to this.

If, on the other hand, it is assumed that the ΔS values calculated from Equation (5), based upon the work of Portalski, are correct, then the varying values of ϵ indicate that the effect of the surface area increase is not constant for all systems. Therefore the values of ϵ should be a function of the system physical properties. Except for the methanol data, these exponents do show a systematic decrease with the Schmidt number.

CONCLUSIONS

It is concluded that:

1. Vaporization data in wetted-wall columns can be correlated by Equation (9). This equation correlated data over temperature ranges of 15° to 80°F.
2. The effect of liquid flow variations can be empirically correlated by Portalski's function of surface area increase as given by Equation (5).

ACKNOWLEDGMENT

Thanks are due to E. I. du Pont de Nemours and Company for a Summer Research Grant held by C. A. Plank and the National Science Foundation for a Cooperative Graduate Fellowship held by J. Robert Reker. Thanks are also due to the Air Reduction Company and the Olin Mathieson Organic Chemicals Division for donation of the helium and ethylene glycol, respectively.

NOTATION

a = interfacial area/unit volume
A = interfacial area
C_G, C_L = specific heat of gas and liquid, respectively
d = tower diameter
D_v = vapor diffusivity
g = acceleration of gravity, cm./sec.^{2}
G = mass flow rate of gas/unit area
h_G = heat transfer coefficient, liquid to gas
k_G = gas phase mass transfer coefficient
L = mass flow rate of liquid
M, M_{avg} = molecular weight of vaporizing liquid and bulk gas stream, respectively
$(\mathrm{MTG}) = N_{Sh}\left(\frac{p_{BM}}{P}\right) N_{ReG}^{-0.83}$
N_{ReG} = gas Reynolds number, dG/μ_G
N_{ReL} = liquid Reynolds number, $4\Gamma/\mu_L$
N_{ScG} = gas Schmidt number, $\mu_G/\rho D_v$
N_{Sh} = Sherwood number, $\frac{k_G RTd}{D_v}$
p, p^* = partial pressure and vapor pressure, respectively, of diffusing material
p_{BM} = logarithmic mean partial pressure of nondiffusing component
P = total pressure
t_G, t_L, t_f = temperature of gas, bulk liquid film, and jacket fluid, respectively
U = heat transfer coefficient from liquid to jacket fluid
W = vaporization rate, mass/time
Z = height

Greek Letters

Γ = mass flow rate of liquid per unit perimeter
ΔS = percent increase in liquid surface area [Equation (5)]
ϵ = exponent on (ΔS) term
λ = latent heat of vaporizing fluid
μ_G = gas viscosity
μ_L = liquid viscosity, poise
ν_L = kinematic liquid viscosity, sq.cm./sec.
ρ = gas density
σ = surface tension, dynes/cm.

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Manuscript received September 29, 1965; revision received March 30, 1966; paper accepted April 4, 1966.